Subthreshold K^+ Production on Nuclei by π^+ Mesons

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ABSTRACT

The inclusive K^+ mesons production in π^+–nucleus reactions in the subthreshold energy regime is analyzed with respect to the one–step ($\pi^+ n \to K^+ \Lambda$) and the two–step ($\pi^+ n \to \eta p_1$, $\eta p_2 \to K^+ \Lambda$) incoherent production processes on the basis of an appropriate folding model, which allows one to take into account the various forms of an internal nucleon momentum distribution as well as on– and off–shell propagation of the struck target nucleon. Contrary to proton–nucleus reactions primary reaction channel is found to be significant practically at all considered energies. Detailed predictions for the K^+ total and invariant differential cross sections from $\pi^+ C^{12}$– and $\pi^+ Pb^{208}$–collisions at subthreshold energies are provided.

Introduction

The extensive investigations of the production of K^+ mesons in proton–nucleus [1–7] and nucleus-nucleus [8–21] reactions at incident energies lower than the free nucleon-nucleon threshold have been carried out in the past years. Because of the high K^+ production threshold (1.58 GeV) in the nucleon-nucleon collision and the rather weak K^+ rescattering in the surrounding medium compared to the pions, etas, antiprotons and antikaons, from these studies one hopes to extract some additional information about the properties of nuclear matter, reaction dynamics, in–medium properties of hadrons at both normal and high nuclear densities. However because of the complexity of collision dynamics and uncertainties in elementary kaon production cross sections close to the production thresholds [17, 22], in spite of large efforts, subthreshold kaon production is still far from being fully understood. To better understand the phenomenon of the subthreshold kaon production in pA– and AA–interactions it is necessary to undertake experimental and theoretical investigation of the subthreshold kaon production in πA–collisions, because in such collisions one may hope to get a clearer insight into the nuclear structure and the production mechanism [1, 23]. It should be noted that at present there is no experimental data on the kaon and antikaon subthreshold production in pion–induced reactions. Theoretical study of the subthreshold K^- production in πA–collisions in the framework of the first–collision model has been performed elsewhere [23]. The aim of the present work is to explore the influence of an internal nucleon momentum distribution on the description of K^+ subthreshold production in $\pi^+ A$–interactions as well as to evaluate the contributions from primary and secondary channels to the K^+ production process. In this study we present the predictions for total and differential K^+ production cross sections from $\pi^+ C^{12}$– and $\pi^+ Pb^{208}$–collisions in the subthreshold regime, obtained using the appropriate folding model for the primary and secondary production processes [23–25].

1.   The Model and Inputs

1.1 Direct K^+ Production Process

Apart from participation in the elastic scattering an incident pion can produce a K^+ directly in the first inelastic πN–collision due to the nucleon Fermi motion. Since we are interested in the far subthreshold region, we have taken into account the following elementary process which requires the least amount of energy and, hence, has the lowest free production threshold (0.76 GeV) [*] :

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[*] Calculations show that the elementary processes with a kaon and a Σ–particle in the final states, opening up at slightly higher energy (0.89 GeV), contribute to the total K^+ production cross section on C^{12} target nucleus with internal shell–model momentum distribution at 700 MeV initial kinetic energy the values of the order of 8% and 1% when the target nucleon is on– and off–shell, respectively. The contributions from these processes and from the primary process (1) to the kaon laboratory momentum spectrum are comparable

$$\pi^+ + n \rightarrow K^+ + \Lambda. \tag{1}$$

Because the kaon rescatterings affect the spectrum mainly at large angles [12, 13, 26, 27] and since we have an intention to calculate the kaon spectrum at zero laboratory angle, where the cross section is the largest, we will neglect the kaon rescatterings in the present study [†] . Then we can represent the invariant inclusive cross section of K^+ production on nuclei by the initial π^+ meson with momentum $\mathbf{p}_0$ as follows [23–25]:

$$E_{K^+} \frac{d\sigma^{(prim)}_{\pi^+ A \rightarrow K^+ X}(\mathbf{p}_0)}{d\mathbf{p}_{K^+}} = I_V[A, \sigma^{tot}_{\pi^+ N}(p_0)] \left\langle E_{K^+} \frac{d\sigma_{\pi^+ n \rightarrow K^+ \Lambda}(\mathbf{p}_0, \mathbf{p}_{K^+})}{d\mathbf{p}_{K^+}} \right\rangle, \tag{2}$$

where

$$I_V[A, \sigma^{tot}_{\pi^+ N}(p_0)] = N \int \rho(\mathbf{r}) d\mathbf{r} \exp\left[-\mu(p_0) \int_{-\infty}^{0} \rho(\mathbf{r} + x\mathbf{\Omega}_0) dx\right], \tag{3}$$

$$\mu(p_0) = \sigma^{tot}_{\pi^+ p}(p_0) Z + \sigma^{tot}_{\pi^+ n}(p_0) N; \tag{4}$$

$$\left\langle E_{K^+} \frac{d\sigma_{\pi^+ n \rightarrow K^+ \Lambda}(\mathbf{p}_0, \mathbf{p}_{K^+})}{d\mathbf{p}_{K^+}} \right\rangle = \int n(\mathbf{p}_t) d\mathbf{p}_t \left[E_{K^+} \frac{d\sigma_{\pi^+ n \rightarrow K^+ \Lambda}(\sqrt{s}, \mathbf{p}_{K^+})}{d\mathbf{p}_{K^+}} \right]. \tag{5}$$

Here, $E_{K^+} d\sigma_{\pi^+ n \rightarrow K^+ \Lambda}(\sqrt{s}, \mathbf{p}_{K^+})/d\mathbf{p}_{K^+}$ is the free invariant inclusive cross section for the K^+ production in reaction (1); $\rho(\mathbf{r})$ and $n(\mathbf{p}_t)$ are the density and ground–state momentum distribution of intranuclear nucleons normalized to unit; $\mathbf{p}_t$ is the internal momentum of the struck target nucleon just before the collision; $\sigma^{tot}_{\pi^+ N}(p_0)$ is the total cross section of free $\pi^+ N$–interaction; Z and N are the numbers of protons and neutrons in the target nucleus (A=N+Z); $\mathbf{\Omega}_0 = \mathbf{p}_0/p_0$; $\mathbf{p}_{K^+}$ and E_{K^+} are the momentum and total energy of a K^+ meson, respectively; s is the $\pi^+ n$ center–of–mass energy squared. The expression for s is:

$$s = (E_0 + E_t)^2 - (\mathbf{p}_0 + \mathbf{p}_t)^2, \tag{6}$$

where E_0 and E_t are the projectile's total energy, given by $E_0 = \sqrt{p_0^2 + m_\pi^2}$, and the struck target nucleon total energy, respectively. In our calculations we will use two formulas for E_t. In the first case we take into account the recoil of the residual nucleus. Then the energy that the struck target nucleon brings into the collision is equal to [3,28]:

$$E_t = M_A - \sqrt{(-\mathbf{p}_t)^2 + M_{A-1}^2}, \tag{7}$$

only at low kaon momenta. But we are interested in the high–momentum part of the kaon spectrum because of its strong sensitivity to the choice of the nucleon momentum distribution

[†] This is allowed in calculating the total kaon yield as kaons cannot be absorbed in nuclear medium due to the strangeness conservation.

where M_A and M_{A-1} are the masses of the initial target nucleus and the recoiling residual nucleus, respectively. It is easily seen that in this case the struck target nucleon is off–shell and for a large target nucleus E_t is approximately equal to the rest mass of nucleon m_N. In the opposite case we assume that the struck target nucleon is on–shell and E_t is given simply by:

$$E_t = \sqrt{p_t^2 + m_N^2}. \tag{8}$$

Taking into account the two–body kinematics of the elementary process (1), we can readily get the following expression for the Lorentz invariant inclusive cross section for this process:

$$E_{K^+} \frac{d\sigma_{\pi^+ n \to K^+ \Lambda}(\sqrt{s}, \mathbf{p}_{K^+})}{d\mathbf{p}_{K^+}} = \frac{\pi}{I_2(s, m_\Lambda, m_K)} \frac{d\sigma_{\pi^+ n \to K^+ \Lambda}(s)}{d \overset{*}{\Omega}} \times$$

$$\times \frac{1}{(\omega_1 + E_t)} \delta \left[\omega_1 + E_t - \sqrt{m_\Lambda^2 + (\mathbf{Q}_1 + \mathbf{p}_t)^2} \right], \tag{9}$$

$$I_2(s, m_\Lambda, m_K) = \frac{\pi}{2} \frac{\lambda(s, m_\Lambda^2, m_K^2)}{s}, \tag{10}$$

$$\lambda(x, y, z) = \sqrt{[x - (\sqrt{y} + \sqrt{z})^2][x - (\sqrt{y} - \sqrt{z})^2]}, \tag{11}$$

$$\omega_1 = E_0 - E_{K^+}, \ E_{K^+} = \sqrt{p_{K^+}^2 + m_K^2}, \ \mathbf{Q}_1 = \mathbf{p}_0 - \mathbf{p}_{K^+}. \tag{12}$$

Here, $d\sigma_{\pi^+ n \to K^+ \Lambda}(s)/d \overset{*}{\Omega}$ is the K^+ differential cross section in the $\pi^+ n$ center–of–mass system; m_Λ, m_K are the rest masses of a Λ hyperon and a kaon. According to Cugnon et al. [29], we choose the K^+ angular distribution in the form, involving only Legendre polynomials up to the first order:

$$\frac{d\sigma_{\pi^+ n \to K^+ \Lambda}(s)}{d \overset{*}{\Omega}} = [1 + A_1(\sqrt{s}) \cos \overset{*}{\vartheta}_{K^+}] \frac{\sigma_{\pi^+ n \to K^+ \Lambda}(\sqrt{s})}{4\pi}. \tag{13}$$

The parameter A_1 and the total cross section $\sigma_{\pi^+ n \to K^+ \Lambda}$ of reaction (1) can be parametrized by [29]:

$$A_1(\sqrt{s}) = \begin{cases} 5.26 \left(\frac{\sqrt{s} - \sqrt{s_0}}{\mathrm{GeV}} \right) & \text{for } \sqrt{s_0} < \sqrt{s} \leq 1.8 \text{ GeV} \\ 1 & \text{for } \sqrt{s} > 1.8 \text{ GeV}, \end{cases} \tag{14}$$

$$\sigma_{\pi^+ n \to K^+ \Lambda}(\sqrt{s}) = \begin{cases} 10.0 \left(\frac{\sqrt{s} - \sqrt{s_0}}{\mathrm{GeV}} \right) \ [\mathrm{mb}] & \text{for } \sqrt{s_0} < \sqrt{s} \leq 1.7 \text{ GeV} \\[2mm] 0.09 \left(\frac{\mathrm{GeV}}{\sqrt{s} - 1.6\mathrm{GeV}} \right) \ [\mathrm{mb}] & \text{for } \sqrt{s} > 1.7 \text{ GeV}, \end{cases} \tag{15}$$

where $\sqrt{s_0} = m_K + m_\Lambda = 1.61$ GeV is the threshold energy.

The internal nucleon momentum distribution, $n(\mathbf{p}_t)$, is a crucial point in the evaluation of the subthreshold production of any particles on a nuclear target. Therefore, we calculated the cross sections for the K^+ production in $\pi^+\text{C}^{12}$– and $\pi^+\text{Pb}^{208}$– collisions using various types of $n(\mathbf{p}_t)$. For K^+ production calculations in the case of C^{12} target nucleus reported here the following forms of $n(\mathbf{p}_t)$ have been used.

The standard shell–model momentum distribution [23, 24]:

$$n(\mathbf{p}_t) = \frac{(b_0/\pi)^{3/2}}{A/4} \left\{ 1 + \left[\frac{A-4}{6} \right] b_0 p_t^2 \right\} \exp\left(-b_0 p_t^2\right), \tag{16}$$

where $b_0 = 68.5(\text{GeV}/c)^{-2}$.

The momentum distribution in which the part corresponding to the $1p_{3/2}$ shell and having the exponential fall–off at high momentum p_t, was inferred by Million [30] from the $(e, e'p)$ and (γ, p) experiments [31]:

$$n(\mathbf{p}_t) = \frac{1}{A/4} \left\{ n_{1/2}(\mathbf{p}_t) + \left[\frac{A-4}{4} \right] n_{3/2}(\mathbf{p}_t) \right\}, \tag{17}$$

where

$$n_{1/2}(\mathbf{p}_t) = (\pi\nu)^{-3/2} \exp\left(-p_t^2/\nu\right), \tag{18}$$

$$n_{3/2}(\mathbf{p}_t) = C_1 n_{HO}(\mathbf{p}_t) + C_2 n_{exp}(\mathbf{p}_t), \tag{19}$$

$$n_{HO}(\mathbf{p}_t) = \frac{2}{3}(\pi\nu)^{-3/2}(p_t^2/\nu) \exp\left(-p_t^2/\nu\right),$$

$$n_{exp}(\mathbf{p}_t) = \left[\frac{1}{24\pi(p_t^0)^3} \right] (p_t/p_t^0) \exp\left(-p_t/p_t^0\right) \tag{20}$$

and $\sqrt{\nu} = 127.0 \ MeV/c$, $p_t^0 = 55.0 \ MeV/c$, $C_1 = 0.997$, $C_2 = 0.003$.

The double Gaussian distribution with a large high–momentum tail extracted by Geaga et al. [32] from high–energy proton backward scattering:

$$n(\mathbf{p}_t) = \frac{1}{(2\pi)^{3/2}(1+\alpha)} \left[\frac{1}{\sigma_1^3} \exp\left(-p_t^2/2\sigma_1^2\right) + \frac{\alpha}{\sigma_2^3} \exp\left(-p_t^2/2\sigma_2^2\right) \right], \tag{21}$$

where $\sigma_1 = 0.119 \ GeV/c$, $\sigma_2 = 0.230 \ GeV/c$. The parameter α which defines the high–momentum part in $n(\mathbf{p}_t)$ is 0.06 for C^{12} and is proportional to $A^{1/3}$ for other target nuclei. It is worth noting that the fractions of nucleons with intranuclear momenta greater than the Fermi momentum $p_F = 250 \ MeV/c$ are of the order of 10%, 13% and 25%, respectively for the distributions (16), (17) and (21). We show in Fig.1 the $n(\mathbf{p}_t)$ results for C^{12}. One can see that the momentum distribution (21) differs significantly from distributions (16) and (17) at $p_t \geq 400 \ MeV/c$.

For K^+ production calculations in the case of $\pi^+\text{Pb}^{208}$–collisions besides the function (21) we used the following three types of $n(\mathbf{p}_t)$.

The completely degenerate Fermi gas momentum distribution:

$$n(\mathbf{p}_t) = \frac{1}{\frac{4}{3}\pi p_F^3}\theta(p_F - p_t), \ \theta(x) = \frac{x + |x|}{2|x|}. \tag{22}$$

Six Gaussian fit to the momentum distribution obtained from density-dependent Hartree-Fock calculations with the Skyrme Hamiltonian [33]:

$$n(\mathbf{p}_t) = \sum_{i=1}^{6} n_i \left(\frac{b_i}{\pi}\right)^{3/2} \exp\left(-b_i p_t^2\right), \tag{23}$$

where $(\sum_{i=1}^{6} n_i = 1)$ $n = \{2.2745, -13.818, 36.808, -40.447, 19.868, -3.6855\}$, $b_i = 32.716 \cdot 1.2804^i \ (\text{GeV/c})^{-2}$.

Moniz's parametrization based on calculations of nucleon–nucleon correlations in the nuclear matter [34]:

$$n(\mathbf{p}_t) = \begin{cases} \left(\frac{3}{4\pi p_F^3}\right)\left(1 - 6\left(\frac{p_F a_1}{\pi}\right)^2\right) & \text{for} \quad 0 < p_t < p_F \\ \left(\frac{3}{4\pi p_F^3}\right)\left(2\left(\frac{p_F a_1}{\pi}\right)^2 \left(\frac{p_F}{p_t}\right)^4 / \left(1 - \frac{p_F a_1}{8}\right)\right) & \text{for} \quad p_F < p_t < 4 \ \text{GeV/c} \\ 0 & \text{for} \quad p_t > 4 \ \text{GeV/c}, \end{cases} \tag{24}$$

with $a_1 = 2(\text{GeV/c})^{-1}$. Use of the assumptions for $n(\mathbf{p}_t)$ presented above enabled us to investigate the sensitivity of the predictions for K^+ cross sections from π^+A–collisions to the high–momentum tail of $n(\mathbf{p}_t)$ at different incident energies.

Consider now the integral (3) which represents the effective number of neutrons for the $\pi^+ n \to K^+ \Lambda$ reaction on nuclei. A simpler expression can be given for $I_V[A, \sigma_{\pi^+ N}^{tot}(p_0)]$, if $\rho(\mathbf{r})$ is a spherical function in the coordinate space [22]:

$$I_V[A, \sigma_{\pi^+ N}^{tot}(p_0)] = \frac{\pi N}{\mu(p_0)} \int_0^\infty db_\perp^2 \left\{1 - \exp\left[-\mu(p_0) \int_{-\infty}^{\infty} \rho\left(\sqrt{b_\perp^2 + t^2}\right) dt\right]\right\}. \tag{25}$$

In particular, for the Gaussian nuclear density $(\rho(\mathbf{r}) = (b/\pi)^{3/2} \exp\left(-br^2\right)$, $b = 0.248$ fm^{-2} for C^{12}) we get:

$$I_V[A, \sigma_{\pi^+ N}^{tot}(p_0)] = \frac{N}{x_G} \int_0^1 \frac{dt}{t}(1 - e^{-x_G t}), \quad x_G = \mu(p_0)b/\pi. \tag{26}$$

Whereas for a nucleus with a uniform density of nucleons of radius R the exact analytic expression for $I_V[A, \sigma_{\pi^+ N}^{tot}(p_0)]$ can be readily obtained [‡] :

$$I_V[A, \sigma_{\pi^+ N}^{tot}(p_0)] = \frac{3N}{a_2^3}\left[\frac{a_2^2}{2} - 1 + (1 + a_2)e^{-a_2}\right], \quad a_2 = \frac{3\mu(p_0)}{2\pi R^2}. \tag{27}$$

[‡] Numerical calculations, carried out in accordance with the formulas (25)–(27), show that the difference between the results obtained for a nucleus with a diffuse boundary and those obtained for a nucleus with a sharp boundary constitutes a value of the order of ten percents.

In our calculations of the K^+ production cross sections on Pb^{208} and C^{12} target nuclei we have used for $I_V[A, \sigma_{\pi^+ N}^{tot}(p_0)]$ the equations (25) and (26), respectively. For the nuclear density $\rho(\mathbf{r})$ in the case of Pb^{208} nucleus we have assumed a Woods–Saxon form $\rho(\mathbf{r}) = \rho_0 / \left\{ 1 + \exp\left[(r - R_{1/2})/a \right] \right\}$ with $R_{1/2} = 1.07 A^{1/3}$ fm, $a = 0.545$ fm [25].

Now let us perform an averaging of the $\pi^+ n \to K^+ \Lambda$ invariant differential cross section (9) over the Fermi motion of the neutrons in the nucleus using the properties of the Dirac δ–function. Choosing the spherical coordinates in $\mathbf{p}_t$ space with z axis parallel to $\mathbf{Q}_1$ and using the advantage of the spherical symmetry of $n(\mathbf{p}_t)$ we get the following formula for the Fermi–averaged cross section (5) after the integration over the angle between $\mathbf{Q}_1$ and $\mathbf{p}_t$ and a few other manipulations:

$$\left\langle E_{K^+} \frac{d\sigma_{\pi^+ n \to K^+ \Lambda}(\mathbf{p}_0, \mathbf{p}_{K^+})}{d\mathbf{p}_{K^+}} \right\rangle = \frac{\pi}{Q_1} \times \tag{28}$$

$$\times \int_{p_t^-}^{p_t^+} p_t dp_t n(p_t) \int_0^{2\pi} d\varphi \frac{1}{I_2[s(x_0, \varphi), m_\Lambda, m_K]} \frac{d\sigma_{\pi^+ n \to K^+ \Lambda}[s(x_0, \varphi)]}{d\overset{*}{\Omega}},$$

where

$$p_t^- = \left| Q_1 \beta + \omega_1 \sqrt{\beta^2 + 4 m_N^2 (Q_1^2 - \omega_1^2)} \right| / 2(Q_1^2 - \omega_1^2), \quad p_t^+ = +\infty, \tag{29}$$

$$Q_1 = |\mathbf{Q}_1|, \quad \beta = m_N^2 - m_\Lambda^2 - (Q_1^2 - \omega_1^2);$$

$$s(x, \varphi) = m_\pi^2 + m_N^2 + 2 E_0 E_t - 2 p_0 p_t (\cos \vartheta_{\mathbf{Q}_1} \cdot x + \sin \vartheta_{\mathbf{Q}_1} \cdot \sqrt{1 - x^2} \cos \varphi), \tag{30}$$

$$x_0 = (\beta + 2\omega_1 E_t)/2 Q_1 p_t, \quad \cos \vartheta_{\mathbf{Q}_1} = \mathbf{p_0 Q_1}/p_0 Q_1, \quad \sin \vartheta_{\mathbf{Q}_1} = \sqrt{1 - \cos^2 \vartheta_{\mathbf{Q}_1}} \tag{31}$$

in the cases when the struck target neutron is assumed to be on-shell ($E_t = \sqrt{p_t^2 + m_N^2}$) and [§] $Q_1^2 - \omega_1^2 > 0$, whereas

$$p_t^\pm = Q_1 \pm \sqrt{(\omega_1 + m_N)^2 - m_\Lambda^2}; \tag{32}$$

$$s(x, \varphi) = m_\pi^2 + m_N^2 + 2 m_N E_0 - p_t^2 - 2 p_0 p_t (\cos \vartheta_{\mathbf{Q}_1} \cdot x + \sin \vartheta_{\mathbf{Q}_1} \cdot \sqrt{1 - x^2} \cos \varphi), \tag{33}$$

$$x_0 = \left[(\omega_1 + m_N)^2 - m_\Lambda^2 - Q_1^2 - p_t^2 \right] / 2 Q_1 p_t, \tag{34}$$

if the struck target neutron is assumed to be off–shell ($E_t = m_N$). It is worth noting that if the kaons escape in the beam direction then the expression (28) for the Fermi–averaged invariant differential cross section of the $\pi^+ n \to K^+ \Lambda$ process is reduced in view of equations (30), (31) and (33) to an integral only over neutron momentum p_t. Therefore, the differential cross section (2) for K^+ production in $\pi^+ A$–reactions in this case is more directly related to the momentum distribution of the neutrons in the ground state than at other outgoing angles. Figure 2 presents

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[§] The main contribution to the K^+ production in $\pi^+ A$–collisions at incident beam energies considered here comes from a region of the kaon energies and momenta, where this condition is satisfied.

our calculations by (29) of the lower limit of the relevant neutron's momenta at 600 and 700 MeV primary–pion energies as a function of the momentum of a kaon produced at an angle of 0^0 in the laboratory frame; and on Figure 3 the results of the similar calculations by (32) for the lower and upper limits of these momenta are shown, when the struck target neutron is assumed to be off–shell. It is clearly seen that in order to produce kaons, for example, at 600 MeV incident pion energy ϵ_0 neutron momenta higher than about 200–250 MeV/c are necessary. Since in this momentum region the distribution function $n(p_t)$ falls rapidly with p_t (see Fig.1), the integration of equation (28) is exhausted within a relatively small range of p_t values above p_t^- – the smallest value of the neutron's momentum p_t which can satisfy the energy conservation condition in (9). One can also see from Figure 2 that the lower is the pion energy, the substantially larger internal momentum is necessary to produce a kaon with a given high momentum in the on–shell struck target neutron case. The allowed kinematical region becomes significantly smaller with decreasing incident energy for the off–shell assumption about the struck target neutron, as it follows from Figure 3.

Finally, performing the integration of formula (2) (divided by E_{K^+}) over the momentum $\mathbf{p}_{K^+}$ we easily get the following expression for the total cross section for K^+ production in π^+A–collisions from the one–step elementary process (1)

$$\sigma^{(prim)}_{\pi^+A \to K^+X}(\mathbf{p}_0) = I_V[A, \sigma^{tot}_{\pi^+N}(p_0)] \int n(\mathbf{p}_t)d\mathbf{p}_t\sigma_{\pi^+n \to K^+\Lambda}(\sqrt{s}), \qquad (35)$$

which is used hereafter.

Let us focus now on the two–step K^+ production mechanism.

1.2 Two–Step K^+ Production Process

Kinematical considerations show that in the bombarding energy range of our interest ($\epsilon_0 \leq 0.76\ GeV$) the following two–step production process may contribute to the K^+ production in π^+A–interactions [¶] . An incident pion can produce in the first inelastic collision with an intranuclear neutron also an η meson through the elementary reaction:

$$\pi^+ + n \to \eta + p. \qquad (36)$$

We remind that the free threshold energy for this reaction is 0.56 GeV. Then the intermediate η meson, which is assumed to be on–shell, produces the kaon on a proton of the target nucleus via the elementary subprocess with the lowest free production threshold (0.20 GeV):

$$\eta + p \to K^+ + \Lambda, \qquad (37)$$

[¶]We have neglected the two–step K^+ production processes with heavier mesons in the intermediate states due to their larger production thresholds in π^+n–collisions. For example, in the case of ω meson the threshold energy is 0.96 GeV.

provided that this subprocess is energetically possible. For instance, the maximum kinetic energy of η meson produced by a pion with the energy $\epsilon_0 = 0.76\ GeV$ on a target neutron at rest is about $0.33\ GeV$. Therefore, for the beam energies considered here, there is a region of eta's energy where the K^+ production process (37) occurs even if the intranuclear proton is at rest. Due to the Fermi motion of the protons the production will be promoted by the target nucleus. It is thus desirable to evaluate the respective K^+ yield. In order to calculate the K^+ production cross section for π^+A–reactions from the secondary eta induced reaction channel (37) we fold the Fermi–averaged differential cross section for the η production in the reaction (36) (denoted by $< d\sigma_{\pi^+n\to\eta p}(\mathbf{p}_0, \mathbf{p}_\eta)/d\mathbf{p}_\eta >$) with the Fermi–averaged invariant differential cross section for K^+ production in this channel (denoted by $< E_{K^+} d\sigma_{\eta p\to K^+\Lambda}(\mathbf{p}_\eta, \mathbf{p}_{K^+})/d\mathbf{p}_{K^+} >$) and the effective number of np pairs per unit of square (denoted by $I_V[A, \sigma^{tot}_{\pi^+N}(p_0), \sigma^{tot}_{\eta N}(p_\eta), \vartheta_\eta]$), i.e.:

$$E_{K^+}\frac{d\sigma^{(sec)}_{\pi^+A\to K^+X}(\mathbf{p}_0)}{d\mathbf{p}_{K^+}} = \int d\mathbf{p}_\eta I_V[A, \sigma^{tot}_{\pi^+N}(p_0), \sigma^{tot}_{\eta N}(p_\eta), \vartheta_\eta]\times \tag{38}$$

$$\times\left\langle\frac{d\sigma_{\pi^+n\to\eta p}(\mathbf{p}_0, \mathbf{p}_\eta)}{d\mathbf{p}_\eta}\right\rangle\left\langle E_{K^+}\frac{d\sigma_{\eta p\to K^+\Lambda}(\mathbf{p}_\eta, \mathbf{p}_{K^+})}{d\mathbf{p}_{K^+}}\right\rangle,$$

where

$$I_V[A, \sigma^{tot}_{\pi^+N}(p_0), \sigma^{tot}_{\eta N}(p_\eta), \vartheta_\eta] = NZ\int\int d\mathbf{r}d\mathbf{r}_1\theta(x_\parallel)\delta^2(\mathbf{x}_\perp)\rho(\mathbf{r})\rho(\mathbf{r}_1)\times \tag{39}$$

$$\times\exp\left[-\mu(p_0)\int_{-\infty}^{0}\rho(\mathbf{r}_1 + x'\mathbf{\Omega}_0)dx' - \mu(p_\eta)\int_{0}^{x_\parallel}\rho(\mathbf{r}_1 + x'\mathbf{\Omega}_\eta)dx'\right],$$

$$\mathbf{r} - \mathbf{r}_1 = x_\parallel\mathbf{\Omega}_\eta + \mathbf{x}_\perp, \mathbf{\Omega}_\eta = \mathbf{p}_\eta/p_\eta, \mu(p_\eta) = A\sigma^{tot}_{\eta N}(p_\eta); \tag{40}$$

$$\left\langle\frac{d\sigma_{\pi^+n\to\eta p}(\mathbf{p}_0, \mathbf{p}_\eta)}{d\mathbf{p}_\eta}\right\rangle = \int n(\mathbf{p}_t)d\mathbf{p}_t\left[\frac{d\sigma_{\pi^+n\to\eta p}(\sqrt{s}, \mathbf{p}_\eta)}{d\mathbf{p}_\eta}\right], \tag{41}$$

$$\left\langle E_{K^+}\frac{d\sigma_{\eta p\to K^+\Lambda}(\mathbf{p}_\eta, \mathbf{p}_{K^+})}{d\mathbf{p}_{K^+}}\right\rangle = \int n(\mathbf{p}_t)d\mathbf{p}_t\left[E_{K^+}\frac{d\sigma_{\eta p\to K^+\Lambda}(\sqrt{s_1}, \mathbf{p}_{K^+})}{d\mathbf{p}_{K^+}}\right] \tag{42}$$

and according to (9)

$$E_\eta\frac{d\sigma_{\pi^+n\to\eta p}(\sqrt{s}, \mathbf{p}_\eta)}{d\mathbf{p}_\eta} = \frac{\pi}{I_2(s, m_\eta, m_N)}\frac{d\sigma_{\pi^+n\to\eta p}(s)}{d\overset{*}{\Omega}}\times \tag{43}$$

$$\times\frac{1}{(\omega_\eta + E_t)}\delta\left[\omega_\eta + E_t - \sqrt{m_N^2 + (\mathbf{Q}_\eta + \mathbf{p}_t)^2}\right],$$

$$\omega_\eta = E_0 - E_\eta, \quad \mathbf{Q}_\eta = \mathbf{p}_0 - \mathbf{p}_\eta;$$

$$E_{K^+}\frac{d\sigma_{\eta p\to K^+\Lambda}(\sqrt{s_1}, \mathbf{p}_{K^+})}{d\mathbf{p}_{K^+}} = \frac{\pi}{I_2(s_1, m_\Lambda, m_K)}\frac{d\sigma_{\eta p\to K^+\Lambda}(s_1)}{d\overset{*}{\Omega}}\times \tag{44}$$

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$$\times \frac{1}{(\omega_2 + E_t)} \delta \left[\omega_2 + E_t - \sqrt{m_\Lambda^2 + (\mathbf{Q}_2 + \mathbf{p}_t)^2} \right],$$

$$\omega_2 = E_\eta - E_{K^+}, \quad \mathbf{Q}_2 = \mathbf{p}_\eta - \mathbf{p}_{K^+}, \quad s_1 = (E_\eta + E_t)^2 - (\mathbf{p}_\eta + \mathbf{p}_t)^2.$$

Here, $d\sigma_{\pi^+ n \to \eta p}/d\overset{*}{\Omega}$ and $d\sigma_{\eta p \to K^+ \Lambda}/d\overset{*}{\Omega}$ are the free differential cross sections of the subprocesses (36) and (37) in the corresponding center–of–mass systems; $\sigma_{\eta N}^{tot}(p_\eta)$ is the total cross section of the free ηN–interaction; $\mathbf{p}_\eta$, E_η and m_η are the momentum, total energy and the rest mass of η meson, respectively, $\cos \vartheta_\eta = \mathbf{\Omega}_0 \mathbf{\Omega}_\eta$. The quantities $\mu(p_0)$, s and $\theta(x)$ are defined above by the equations (4), (6) and (22), respectively. In our calculations the angular distributions $d\sigma_{\pi^+ n \to \eta p}/d\overset{*}{\Omega}$ and $d\sigma_{\eta p \to K^+ \Lambda}/d\overset{*}{\Omega}$ are assumed to be isotropic. For the total cross section $\sigma_{\pi^+ n \to \eta p}$ of the reaction (36) we used the parametrization suggested in [35], assuming the isospin invariance of the strong interaction ($\sigma_{\pi^+ n \to \eta p} = \sigma_{\pi^- p \to \eta n}$):

$$\sigma_{\pi^+ n \to \eta p}(\sqrt{s}) = \begin{cases} 13.07 \left(\frac{\sqrt{s} - \sqrt{\tilde{s}_0}}{\text{GeV}} \right)^{0.5288} \ [\text{mb}] & \text{for } \sqrt{\tilde{s}_0} < \sqrt{s} \leq 1.589 \ GeV \\[3mm] 0.1449 \left(\frac{\text{GeV}}{\sqrt{s} - \sqrt{\tilde{s}_0}} \right)^{1.452} \ [\text{mb}] & \text{for } \sqrt{s} > 1.1589 \ GeV, \end{cases} \tag{45}$$

where $\sqrt{\tilde{s}_0} = 1.486 \ GeV$ is the threshold energy. Because of the lack of knowledge [||] about the total cross section $\sigma_{\eta p \to K^+ \Lambda}$ of the elementary process (37), to estimate this cross section we choose in this work the following natural way, which was used also in [37] for the evaluation of lambda production cross section in the reaction $\omega N \to K \Lambda$. The probability for producing a kaon in the reaction under consideration is given by the ratio of the $\sigma_{\eta p \to K^+ \Lambda}$ to the ηp–inelastic cross section $\sigma_{\eta p}^{in}$. We assume that this ratio is equal to that of the $\sigma_{\pi^+ n \to K^+ \Lambda}$, defined by (15), to the $\pi^+ n$–inelastic cross section $\sigma_{\pi^+ n}^{in}$ at the same invariant energy $\sqrt{s_1}$. Taking into account that $\sigma_{\eta p}^{in} \approx \sigma_{\pi^+ n}^{in} \approx 20$ mb [25, 38] in the eta and pion energy ranges of interest, we get:

$$\sigma_{\eta p \to K^+ \Lambda}(\sqrt{s_1}) = \sigma_{\pi^+ n \to K^+ \Lambda}(\sqrt{s_1}), \tag{46}$$

which is used hereafter to calculate the K^+ yield in $\pi^+ A$–collisions from the secondary channel (37).

Let us now simplify the expression (38) for the invariant differential cross section for K^+ production in $\pi^+ A$–interactions from the two–step process. Taking into account that the main contribution to the K^+ production comes from fast etas moving in the beam direction and that the ηN total cross section $\sigma_{\eta N}^{tot}$ in the energy

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[||] We know only one publication [36] in which the authors basing on the SU(3)–symmetry of the strong interaction obtained the following relation: $\sigma_{\eta p \to K^+ \Lambda} = 9/2 \sigma_{\bar{K}^0 p \to \Sigma^0 \pi^+}$. However, the use of this relation in the near threshold energy region leads to very large contribution from the elementary process (37) to the total K^+ production cross section in $\pi^+ A$–reactions, which is doubtful from our point of view.

region of interest is approximately constant with a magnitude of $< \sigma_{\eta N}^{tot} > \approx 35$ mb [25,35], we have:

$$E_{K^+} \frac{d\sigma_{\pi^+ A \to K^+ X}^{(sec)}(\mathbf{p}_0)}{d\mathbf{p}_{K^+}} = I_V[A, \sigma_{\pi^+ N}^{tot}, < \sigma_{\eta N}^{tot} >, 0^0] \times \tag{47}$$

$$\times \int\limits_0^{\sqrt{E_0^2 - m_\eta^2}} p_\eta^2 dp_\eta \left\langle \frac{d\sigma_{\pi^+ n \to \eta p}(\mathbf{p}_0, p_\eta, 0^0)}{d\mathbf{p}_\eta} \right\rangle \left\langle E_{K^+} \frac{d\sigma_{\eta p \to K^+ \Lambda}(p_\eta \mathbf{\Omega}_0, \mathbf{p}_{K^+})}{d\mathbf{p}_{K^+}} \right\rangle,$$

where $< d\sigma_{\pi^+ n \to \eta p}(\mathbf{p}_0, p_\eta, 0^0)/d\mathbf{p}_\eta >$ is the corresponding spectrum of etas at an angle of $\vartheta_\eta = 0^0$. Using the way analogous to that used in the derivation of equation (28) as well as after some algebra we can obtain:

$$\left\langle \frac{d\sigma_{\pi^+ n \to \eta p}(\mathbf{p}_0, p_\eta, 0^0)}{d\mathbf{p}_\eta} \right\rangle = \frac{\pi}{2E_\eta Q_\eta} \int\limits_{p_G^-}^{p_G^+} p_t dp_t n(p_t) \frac{\sigma_{\pi^+ n \to \eta p}[\sqrt{s(y_0)}]}{I_2[s(y_0), m_\eta, m_N]}, \tag{48}$$

where

$$p_G^- = \left| \frac{1}{2}Q_\eta - \omega_\eta \sqrt{\frac{1}{4} + \frac{m_N^2}{Q_\eta^2 - \omega_\eta^2}} \right|, \quad p_G^+ = +\infty, \tag{49}$$

$$s(y) = m_\pi^2 + m_N^2 + 2E_0 E_t - 2p_0 p_t y, \quad y_0 = (\omega_\eta^2 - Q_\eta^2 + 2\omega_\eta E_t)/(2Q_\eta p_t)$$

in the case when the struck target nucleon is assumed to be on–shell and

$$p_G^\pm = \left| Q_\eta \pm \sqrt{\omega_\eta^2 + 2m_N \omega_\eta} \right|, \tag{50}$$

$$s(y) = m_\pi^2 + m_N^2 + 2m_N E_0 - p_t^2 - 2p_0 p_t y, \quad y_0 = (\omega_\eta^2 + 2m_N \omega_\eta - Q_\eta^2 - p_t^2)/(2Q_\eta p_t)$$

if the struck target nucleon is off–shell. The quantity $< E_{K^+} \frac{d\sigma_{\eta p \to K^+ \Lambda}(p_\eta \mathbf{\Omega}_0, \mathbf{p}_{K^+})}{d\mathbf{p}_K^+} >$ in (47) is determined by the formulas (28)–(34) in which we have to make the following substitutions:

$$d\sigma_{\pi^+ n \to K^+ \Lambda}/d\overset{*}{\Omega} \to d\sigma_{\eta p \to K^+ \Lambda}/d\overset{*}{\Omega},$$

$$s \to s_1,$$

$$\mathbf{Q}_1, \omega_1 \to \mathbf{Q}_2, \omega_2,$$

$$E_0, \mathbf{p}_0 \to E_\eta, p_\eta \mathbf{\Omega}_0$$

$$m_\pi \to m_\eta$$

and to take into account that for on–shell assumption about the struck target proton if $Q_2^2 - \omega_2^2 < 0$ (this case is realized here) then the lower and upper limits in (28) are:

$$p_t^\pm = \left| Q_2 \beta \mp \omega_2 \sqrt{\beta^2 + 4m_N^2(Q_2^2 - \omega_2^2)} \right| /2(\omega_2^2 - Q_2^2). \tag{51}$$

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One can show that the expression for $I_V[A, \sigma^{tot}_{\pi^+N}(p_0), <\sigma^{tot}_{\eta N}>, 0^0]$ in the case of a nucleus of a radius $R = 1.3A^{1/3}$ fm with a sharp boundary has the following simple form:

$$I_V[A, \sigma^{tot}_{\pi^+N}(p_0), <\sigma^{tot}_{\eta N}>, 0^0] = \frac{9NZ}{2\pi R^2(a_2 - a_3)} \times \tag{52}$$

$$\times \left\{ \frac{1}{a_2^3}[1 - (1 + a_2)e^{-a_2}] - \frac{1}{a_3^3}[1 - (1 + a_3)e^{-a_3}] - \frac{1}{2a_2} + \frac{1}{2a_3} \right\},$$

where $a_3 = 3A <\sigma^{tot}_{\eta N}> /2\pi R^2$ and the quantity a_2 is determined by the formula (27).

Besides of the differential cross section it is of further interest to get the corresponding expression for the total cross section for K^+ production in π^+A–reactions from the two–step process. Integrating the formula (47) (divided by E_{K^+}), we readily obtain

$$\sigma^{(sec)}_{\pi^+A\to K^+X}(\mathbf{p}_0) = I_V[A, \sigma^{tot}_{\pi^+N}(p_0), <\sigma^{tot}_{\eta N}>, 0^0] \times \tag{53}$$

$$\times \int\limits_0^{\sqrt{E_0^2 - m_\eta^2}} p_\eta^2 dp_\eta < \frac{d\sigma_{\pi^+n\to\eta p}(\mathbf{p}_0, p_\eta, 0^0)}{d\mathbf{p}_\eta} > \int n(\mathbf{p}_t)d\mathbf{p}_t \sigma_{\eta p\to K^+\Lambda}(\sqrt{s_1}).$$

Now let us discuss the results of our calculations in the framework of the approach outlined above.

2. Results and Discussion

The expected total cross sections for K^+ production in $\pi^+ + C^{12}$ –reactions from the primary $\pi^+ n \to K^+\Lambda$ and secondary $\eta p \to K^+\Lambda$ channels calculated according to (6)–(8), (15)–(21), (26), (35), (45), (46), (48)–(50), (52), (53) are shown in Fig.4 as functions of the laboratory energy ϵ_0 of the pion. The elementary cross sections $\sigma^{tot}_{\pi^+N}$ in the calculations were borrowed from [38]. It is seen that the η induced production channel becomes comparable to the π^+n channel only at very low energies ($\epsilon_0 < 600$ MeV) if we adopt the off–shell assumption about the struck target nucleon as well as use the nucleon momentum distributions (16), (17) without a large high–momentum tails. The cross section of the K^+ production from the primary π^+n production channel in the energy region of $600 \le \epsilon_0 \le 650$ MeV, where this channel dominates, still strongly depends both on the choice of the nucleon momentum distribution and on the bombarding energy ϵ_0 (contrary to the secondary production channel). The cross section calculated with the momentum distribution (21), extracted from high–energy proton backward scattering, is larger by a factor of 2–10 than that calculated with the shell–model momentum distribution (16) in this region. The momentum distribution (17), deduced from the $(e, e'p)$ and (γ, p) experiments, gives practically the same results as the shell–model momentum distribution (16). The values of the total production cross section in the energy region under consideration lie in

the range of 0.5–30 μb. Such rapid energy dependence of the (π^+, K^+) total cross section in the energy region considered here is a characteristic signature of the $\pi^+ n \to K^+ \Lambda$ one–step production mechanism. Therefore, the measurement of the total K^+ production cross section at incident energies between 600 and 650 MeV seems to be quite promising to study both the kaon production mechanism and the high–momentum components within nucleus. However, it is important to emphasize that in order to get clearer insight into the relative role of the primary and secondary reaction channels, further theoretical efforts are necessary for a better understanding about the η induced elementary K^+ production process (37).

Figure 5 presents the results of similar calculations, using the momentum distributions (21)–(24), for the total cross section for K^+ production in $\pi^+ + Pb^{208}$–reactions. It is seen that the contribution from the two–step production process is comparable to that from the one–step production process, as well as in the previous case, only at energies $\epsilon_0 < 600~MeV$. In order to demonstrate the sensitivity of the kaon yield from the two–step production process to η absorption in the nuclear medium, in Fig. 5 we show the results of calculations carried out under the assumption that the total cross section of ηN–interaction is replaced by the total inelastic cross section of this interaction with a magnitude of 20 mb. We can observe an increase of the K^+ yield by a factor of 1.5 for this target nucleus. It is also seen that the cross section for K^+ production in the far subthreshold region ($600 \le \epsilon_0 \le 650~MeV$) in this case is larger by a factor of 10 than that calculated for $\pi^+ + C^{12}$–reaction. One can see further that for the one–step production process (1) to occur at the bombarding energies $\epsilon_0 < 600~MeV$, the internal momenta greater than 250 MeV/c are needed, if the struck target neutron is assumed to be off-shell. This finding is in agreement with that of Figure 3. It is interesting to note that the nucleon momentum distributions (21) and (24) having the different forms of a high–momentum tail give the close results for the K^+ total cross section from the primary production channel. Therefore, in order to differentiate between these distributions one needs, as it follows from the expression (28), to obtain information about the differential cross sections of the K^+ mesons produced in the reactions under consideration.

In Figs.6 and 7 we show the results of our calculations by (2)–(26), (28)–(34), (38)–(52) of the inclusive invariant cross sections for the kaon production from the primary $\pi^+ n$– and secondary ηp–reaction channels at an angle of 0^0 in the interaction of pions with the energies of 600 MeV (lower lines) and 700 MeV (upper lines) with C^{12} and Pb^{208} nuclei. It is clearly seen that the secondary production channel practically does not contribute to the spectrum of emitted kaons at incident energies between 600 and 700 MeV. Also one can see that the high–momentum part of the spectrum of kaons from the one–step production process (1) is essentially determined by the nucleon momentum distribution, whereas its low–momentum part is not sensitively affected by the choice of the internal momentum distribution in the case of the on–shell assumption about the struck target neutron. The difference between the calculations with allowance for the high–momentum components in the target nu-

cleus and without it becomes larger at lower incident energies also in the case when the struck target neutron is off–shell. Taking into account the considered above, we conclude that the measurement of the total and differential K^+ production cross sections at incident energies between 600 and 700 MeV offers the possibility to check the dominant role of the one–step production process in the subthreshold K^+ production as well as to study the high–momentum components within target nucleus.

3. Summary

In this paper we have calculated the total and differential cross sections for K^+ production from $\pi^+ + C^{12}$– and $\pi^+ + Pb^{208}$–reactions in the subthreshold regime by considering incoherent primary pion–neutron and secondary eta–proton production processes within the framework of an appropriate folding model. This approach takes into account the various forms of an internal nucleon momentum distribution as well as on– and off–shell propagation of the struck target nucleon. It was shown that, contrary to proton–nucleus reactions, the one–step K^+ production mechanism clearly dominates at all subthreshold energies if the struck target nucleon is assumed to be on–shell, whereas for the off–shell assumption about the struck target nucleon the one–step and the two–step reaction mechanisms are of equal importance only at very low energies ($\epsilon_0 < 600$ MeV) in the case of use of the nucleon momentum distributions without a large high–momentum tails. We predict, in particular, a rapid energy dependence of the total cross section for K^+ production from the primary channel as well as its strong sensitivity to the choice of the internal nucleon momentum distribution in the energy domain $\epsilon_0 \approx 600 \div 650$ MeV, where this channel dominates. Such behaviour of the (π^+, K^+) total cross section in this energy domain is a principal signature of the $\pi^+ n \to K^+ \Lambda$ production mechanism. Therefore, from measurements of the inclusive K^+ production cross sections at incident pion energies indicated above one may hopes to obtain a clear information both on the production mechanism and on the high–momentum components within target nucleus.

Acknowledgements

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Figure captions

Fig.1. Momentum distribution for C^{12}. The solid line is the harmonic oscillator momentum distribution (16). The dot–dashed line is the momentum distribution (17), extracted from the $(e, e'p)$ ($p_t < 250\ MeV/c$) and (γ, p) ($p_t >250\ MeV/c$) experiments [31]. The dashed line is the momentum distribution (21) inferred from high–energy proton backward scattering [32].

Fig.2. Lower limit of the internal neutron momentum as a function of the momentum of a kaon produced in beam direction at 600 and 700 MeV incident pion energies in the case when the target neutron is assumed to be on–shell.

Fig.3. Lower and upper limits of the internal neutron momentum as a function of the momentum of a kaon produced in beam direction at 600 and 700 MeV incident pion energies in the case when the target neutron is assumed to be off–shell.

Fig.4. The total cross sections for K^+ production in $\pi^+ + C^{12}$–reactions from primary $\pi^+ n \rightarrow K^+ \Lambda$ and secondary $\eta p \rightarrow K^+ \Lambda$ channels as functions of the laboratory energy of the pion. The cross sections from primary $\pi^+ n$–collisions: the heavy and the light solid lines are calculations with the shell–model momentum distribution (16) with off– and on–shell assumptions about the struck target neutron, respectively; the dot–dashed line is the calculation with the momentum distribution (17) and off–shell struck target neutron; the short– and the long–dashed lines are calculations with distribution (21) with large high–momentum tail with off– and on–shell assumptions about the struck target neutron, respectively. The cross sections from secondary ηp–collisions: the lower and upper dotted lines are calculations with the shell–model momentum distribution (16) with off– and on–shell assumptions about the struck target nucleons, respectively; the short– and the long–dashed lines with two dots are calculations with distribution (21) with off– and on–shell assumptions about the struck target nucleons, respectively. The arrows show the production thresholds on a free neutron and on off–shell neutron with the Fermi momentum of 250 MeV/c and the absolute production threshold.

Fig.5. The calculated total cross sections for K^+ production in $\pi^+ + Pb^{208}$–reactions from primary $\pi^+ n$– and secondary ηp– collisions as functions of the laboratory energy of the pion. The cross sections from primary $\pi^+ n$–collisions: the heavy and the light solid lines are calculations with the Fermi gas momentum distribution (22) with off– and on–shell assumptions about the struck target neutron, respectively; the short–long–dashed line is the calculation with six Gaussian distribution (23) and off–shell struck target neutron; the short– and the long–dashed lines with one dot are calculations with the Moniz's parametrization (24) with off– and on–shell assumptions about the struck target neutron, respectively. The cross sections from

secondary ηp–collisions: the lower and upper dotted lines are calculations with the Fermi gas momentum distribution (22) with off– and on–shell assumptions about the struck target nucleons, respectively; the crosses denote the same as lower dotted line, but it is supposed that the total cross section of ηN–interaction is replaced by its total inelastic cross section with a magnitude of 20 mb. The rest of the notation is the same as in Figure 4.

Fig.6. The inclusive invariant cross sections for the production of K^+ mesons in primary $\pi^+ n$– and secondary ηp–collisions at an angle of 0^0 as functions of the kaon momentum in the interaction of pions with the energies of 600 and 700 MeV with C^{12} nuclei. The dot–dashed line denotes the same as in Figure 4, but it is supposed that the struck target neutron is on–shell. The rest of the notation is the same as in Figure 4.

Fig.7. The inclusive invariant cross sections for the production of K^+ mesons in primary $\pi^+ n$– and secondary ηp–collisions at an angle of 0^0 as functions of the kaon momentum in the interaction of pions with the energies of 600 and 700 MeV with Pb^{208} nuclei. The short–short–long–dashed line denotes here the same as the short–long–dashed line, but it is supposed that the struck target neutron is on–shell. The rest of the notation is the same as in Figure 5.